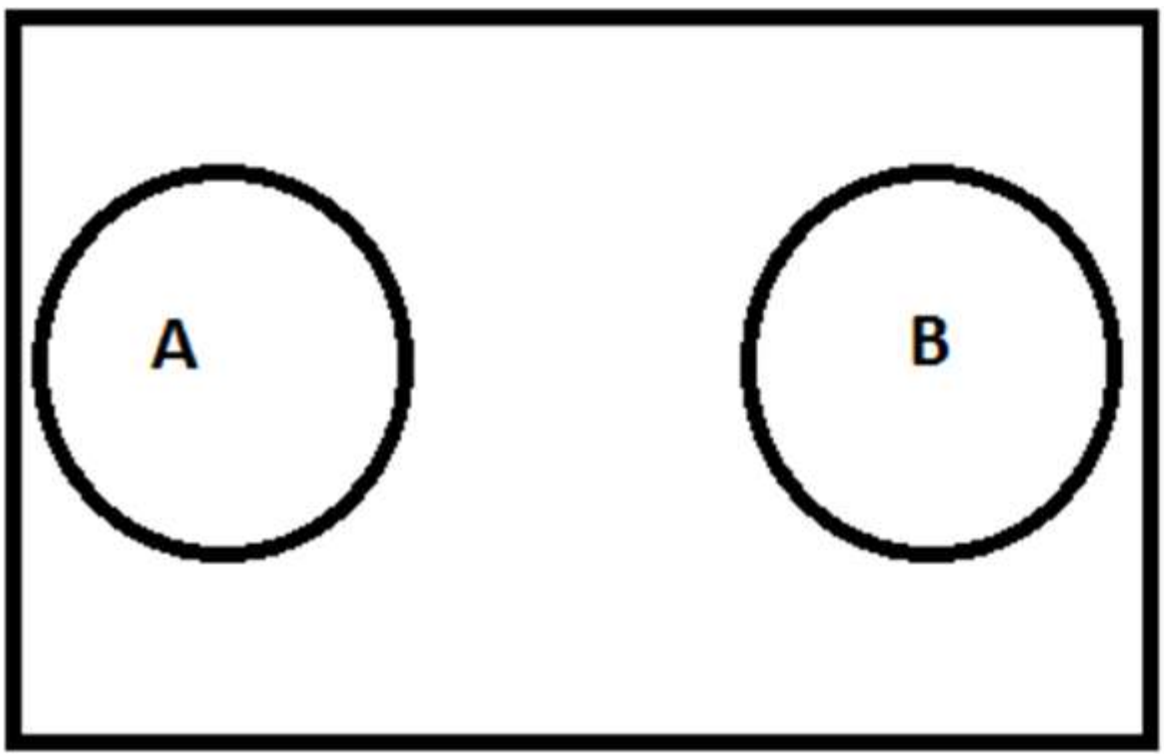
Stat 201: Introduction to Statistics

Standard 14: Probability – Disjoint and Independent Events

Adjectives for Events: Mutually Exclusive (Disjoint)

- Two events are **mutually exclusive (disjoint)** if they do not have any common outcomes

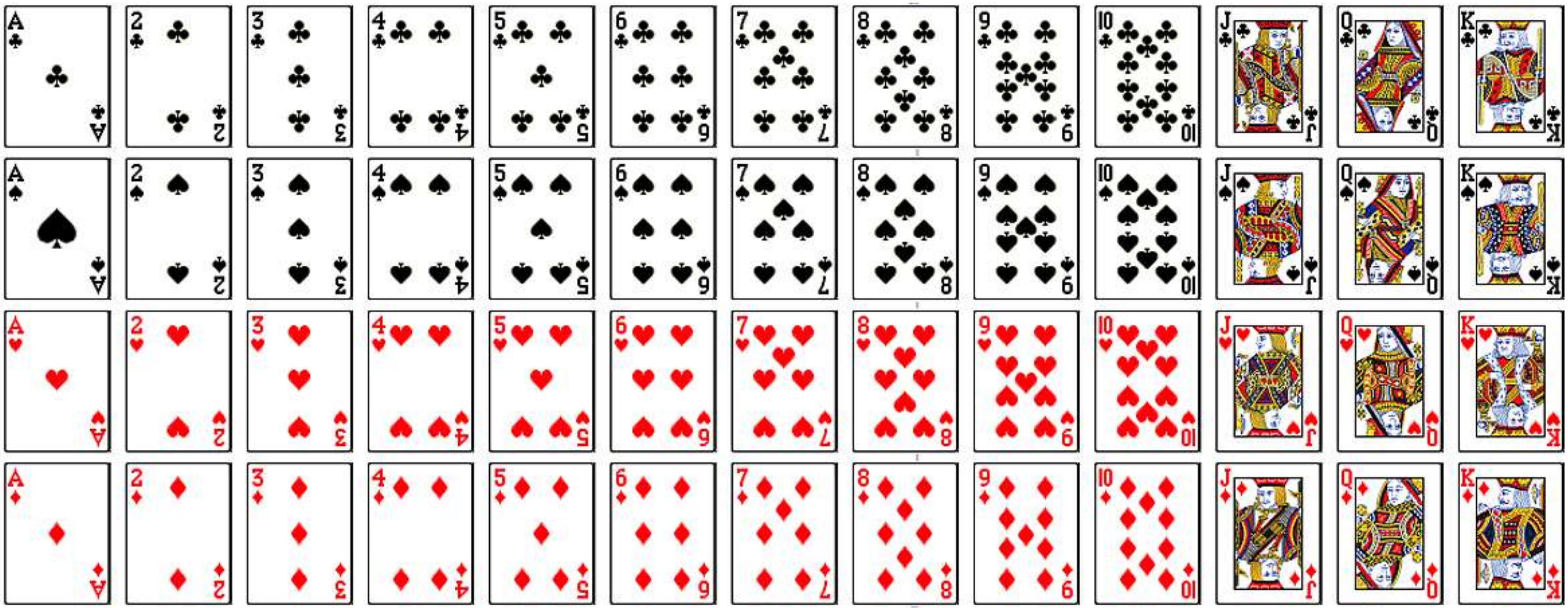


Probability: Additive Rule of Probability for Mutually Exclusive Events

- For mutually exclusive A and B we know that $P(A\ and\ B)$=$P(A \cap B) = 0$

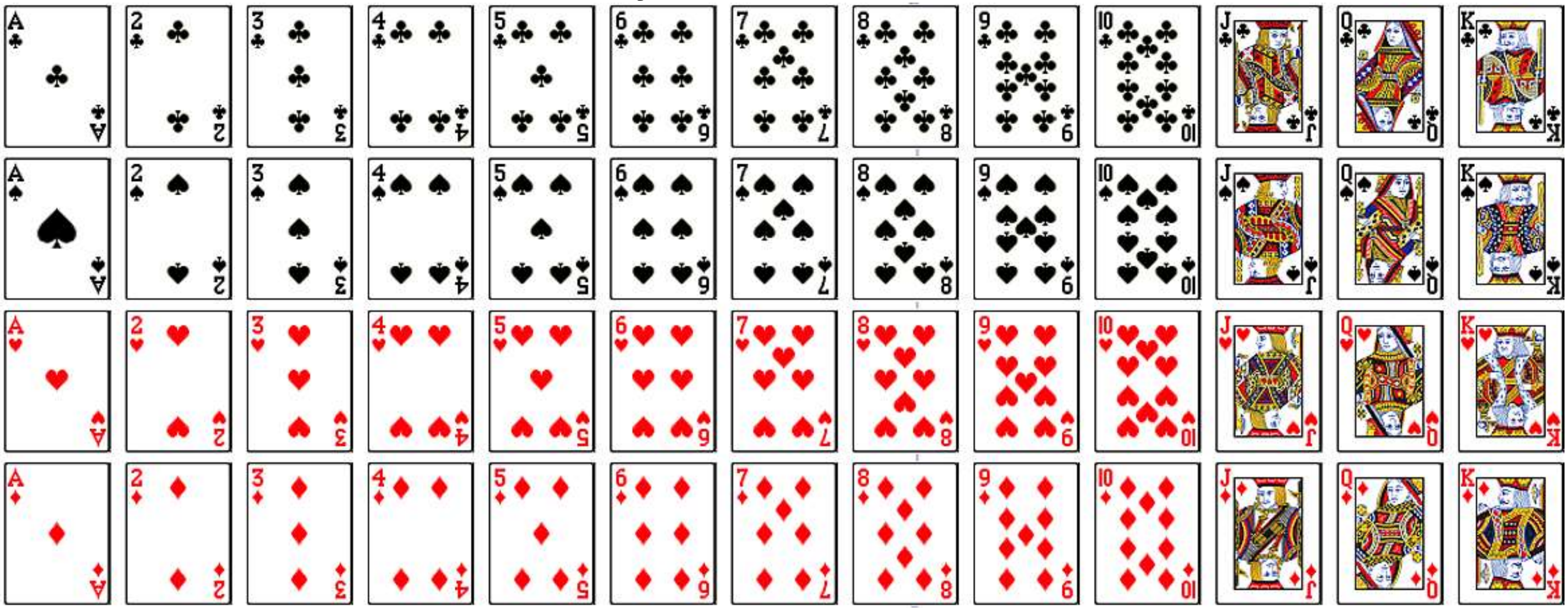
- Therefore the additive rule becomes:
- $P(A\ or\ B) = P(A) + P(B)$
- $P(A \cup B) = P(A) + P(B)$

Mutually Exclusive Events



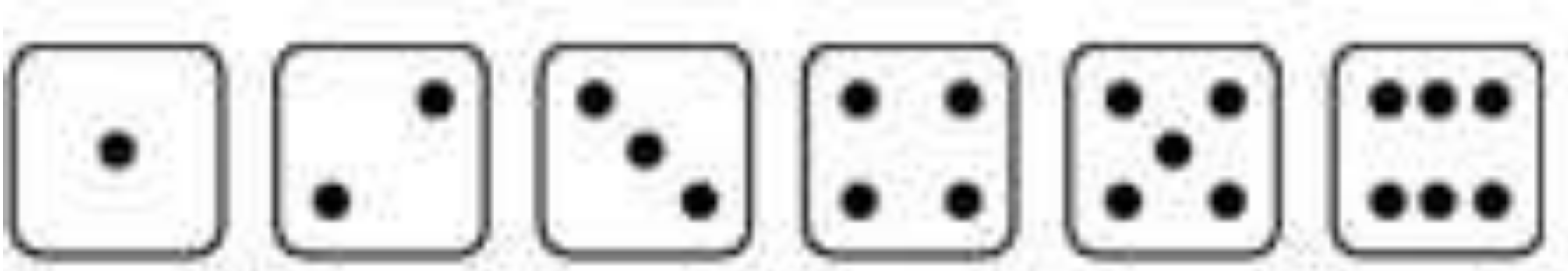
- A= A red card
- B= A black card
- Events A and B are mutually exclusive

Mutually Exclusive Events



- A= A face card
- B= A black car
- Events A and B are **not** mutually exclusive because we can pull out one card that is both black and a face card, like the king of spades

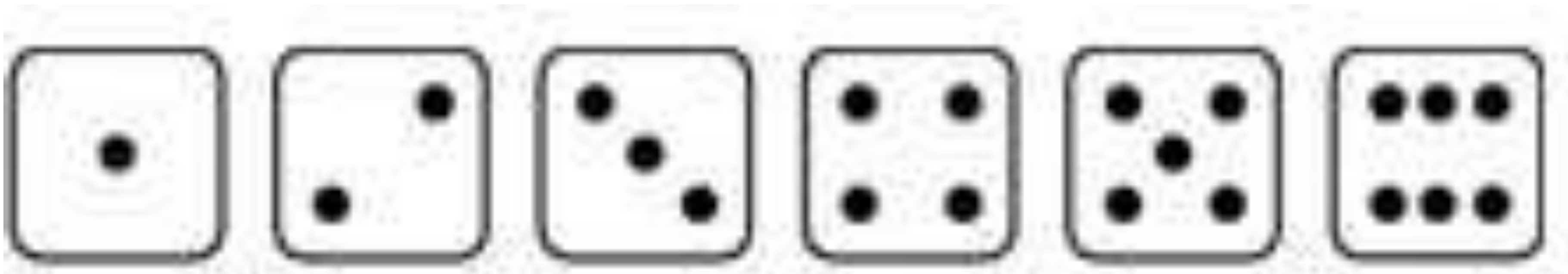
Mutually Exclusive Events



- A= An even side is up
- B= An odd side is up

- Events A and B are mutually exclusive because none of the outcomes are both even and odd

Mutually Exclusive Events



- A= A side that is a multiple of three is up
- B= An even side is up

- Events A and B are **<u>not</u>** mutually exclusive because I can roll a six which is **both** a multiple of three and an even number.

Independence

- Two events, A and B, are **independent** if the fact that A occurs does not affect the probability of B occurring.

- The idea is that knowing the outcome of one of these events doesn't give us any information about the probability of the second event.

Independence

- **Random Experiment:** Flipping a coin twice
- **Events: A=** first toss , **B=** second toss
  - **Q:** If I know that the first toss was heads, i.e. A=heads, does the probability of flipping a heads on the second flip change?

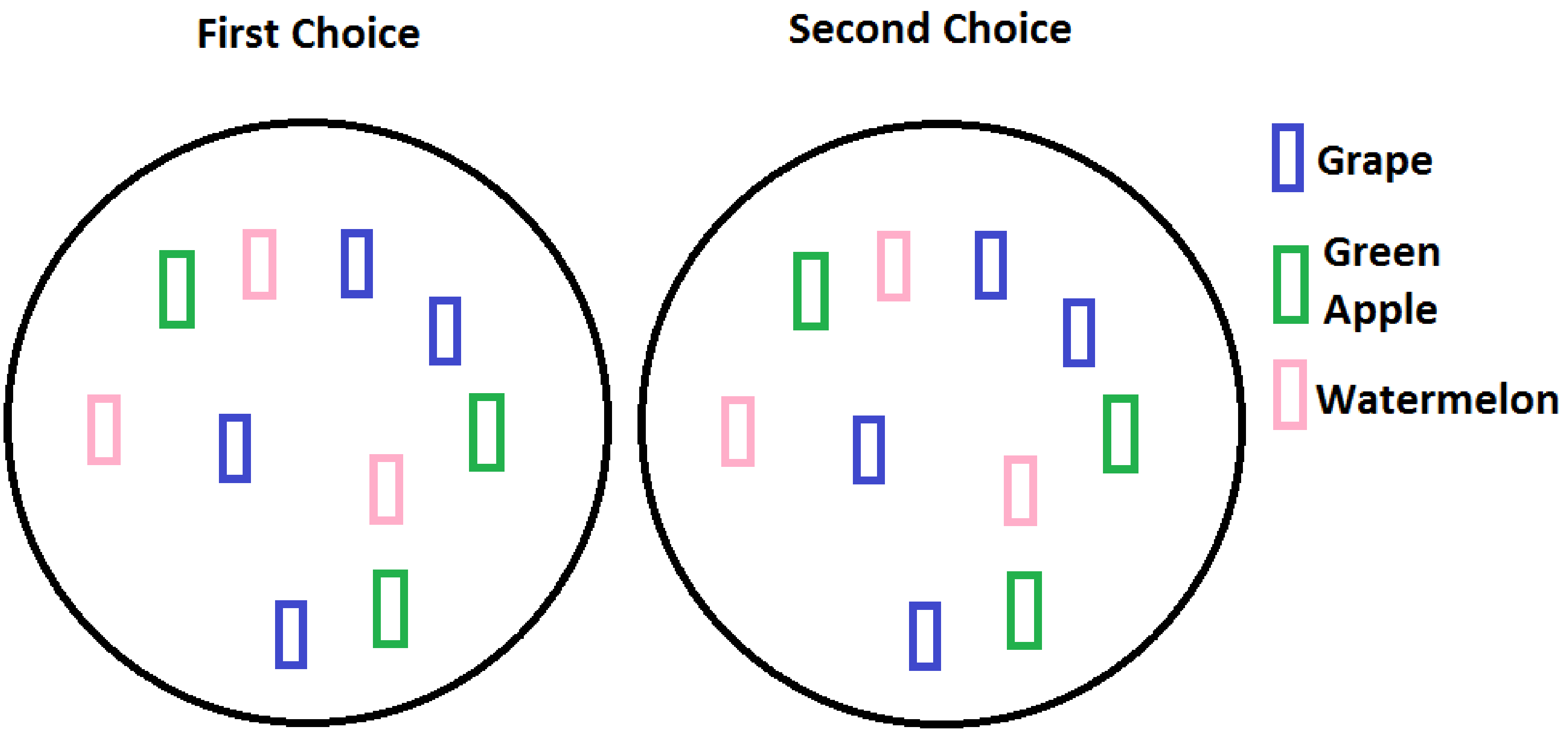
  - **A:** No, the first toss has no impact on the second toss so they are **independent**

Independence

- **Random Experiment:** Picking from different flavors of Jolly Ranchers out of a bowl
- **Events: A=** first pick, **B=** second pick
  - **Q:** If I know that the first pick was watermelon, i.e. A = watermelon, does the probability of picking a watermelon on the second pick change?

  - **A1(with replacement)**: No, because we put back the first choice and we have the same exact bowl to pick our second. Now, there are the same number of watermelon Jolly Ranchers and jolly ranchers overall. This experiment would be **independent.**
    - Math if we started with ten Jolly Ranchers, three of which were watermelon: $P(A=W)=3/10=.3 \rightarrow P(B=W|A=W)=3/10=.3$

Independence

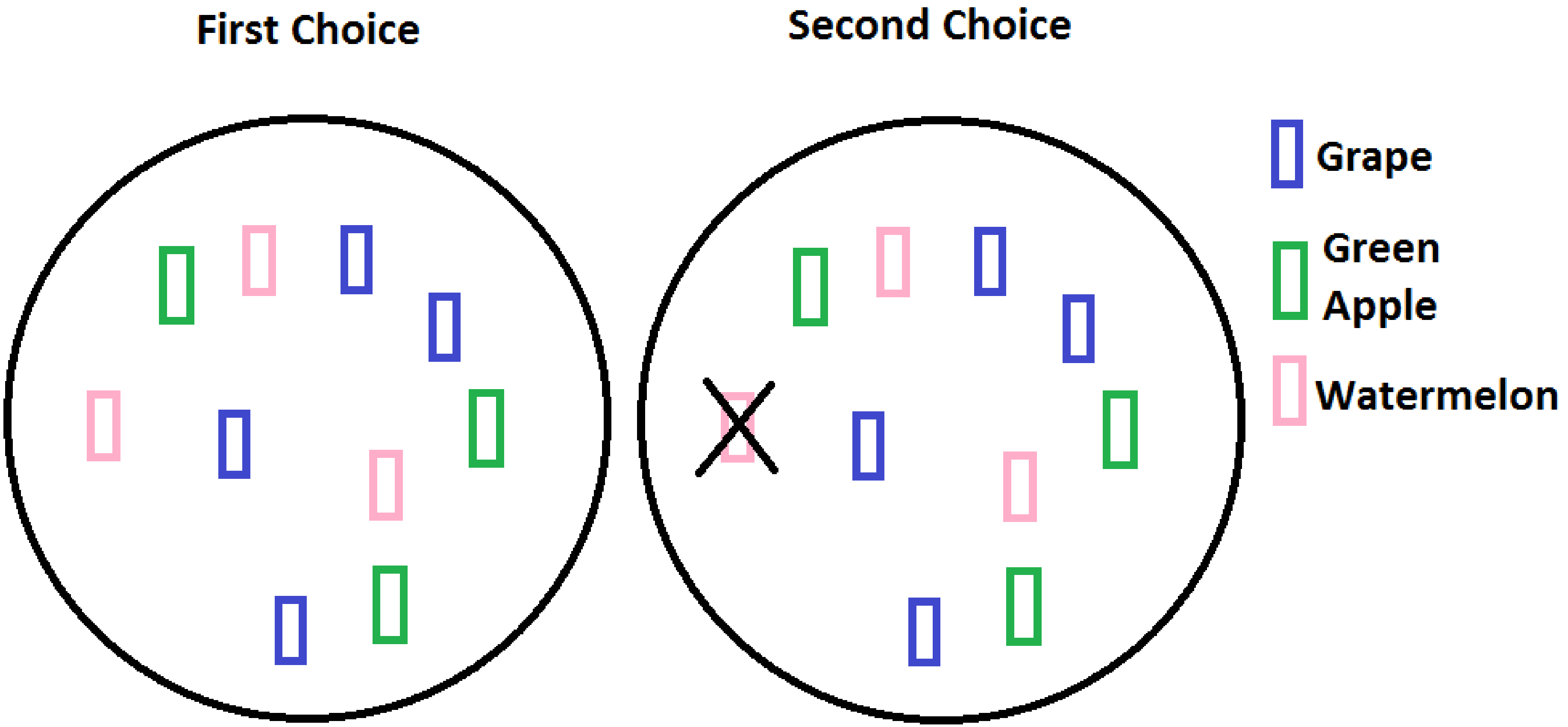


Independence

- **Random Experiment:** Picking from different flavors of Jolly Ranchers out of a bowl
- **Events: A=** first pick, **B=** second pick
  - **Q:** If I know that the first pick was watermelon, i.e. A = watermelon, does the probability of picking a watermelon on the second pick change?

  - **A1(without replacement)**: Yes, because now there are less watermelon Jolly Ranchers and less jolly ranchers overall. This experiment would not be **independent**, it would be **dependent**.
    - Math if we started with ten Jolly Ranchers, three of which were watermelon: $P(A=W)=3/10=.3 \rightarrow P(B=W|A=W)=2/9=.22$

Independence



Walkthrough

- Sampling with or without Replacement*
  - https://www.youtube.com/watch?v=uKTjh-6PFjo

- Conditional Probabilities*
  - https://www.youtube.com/watch?v=JGeTcRfKgBo

To Check for Independence

- Two events, A and B, are independent if:
  1. If P(A|B) = P(A)
  2. If P(B|A) = P(B)
  3. If P(A and B) = P(A)*P(B)

$$\mathrm{P}(\mathrm{A} \cap B) = \mathrm{P}(\mathrm{A}) * \mathrm{P}(B)$$

-**Note:** If any of these are true, the others are also true and the events A and B are independent

Independence

- Recall:
- $\mathrm{P(A\ and\ B)} = \mathrm{P}(\mathrm{A} \cap B) = \mathrm{P(A)} * \mathrm{P(B|A)}$
  $= \mathrm{P(B)} * \mathrm{P}(\mathrm{A}|B)$
  - The probability of A and B happening is either:
    - The probability of A times the probability of B given A
    - The probability of B times the probability of A given B
  - **For independent events**
    - $P(A\ and\ B) = \mathrm{P}(\mathrm{A} \cap B) = P(A) * P(B)$
    - This is because $\mathrm{P(B|A)}$=P(B) and $\mathrm{P}(A|B)$=P(A) when A and B are independent